Abstracts For Southern New England Conference on Quadratic Forms and Modular Forms

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Mod p homology of GL($n, \mathbb{Z}$)

Avner Ash

Let G be a subgroup of finite index in $\mathrm{GL}(n, \mathbb{Z})$, F a finite field of characteristic p, and M an $F[G]$-module, finite dimensional over F, through which G acts via reduction modulo pN, for some integer N prime to p. The group homology $H_i(G, M)$ contains a lot of number theory, which we access through the action of Hecke operators on it. In particular, Scholze has proved that Hecke eigenpackets in the homology have associated Galois representations. I will review these ideas, with an overview of how to compute the homology together with the Hecke action, and a survey of the current state of research and conjecture, to the extent possible in the allotted time.

Generating weights for modules of vector-valued modular forms

Luca Candelori

Given an n-dimensional representation of the metaplectic group (e.g. the Weil representation of a finite quadratic module) we study the module of vector-valued modular forms for this representation, using methods from algebraic geometry. We prove that this module is free of rank n over the ring of level one modular forms, and we discuss the problem of finding the weights of a generating set. For Weil representations of cyclic quadratic modules of order 2p, p a prime, we show how the generating weights can be expressed in terms of class numbers of quadratic imaginary fields. This is joint work with Cameron Franc and Gene Kopp (U. Michigan).

Integral Quadratic Forms and Lattices with Regularity Properties

Andrew G. Earnest

In a seminal paper appearing in 1927, L.E. Dickson introduced the term regular to refer to a positive definite integral ternary quadratic form with the property that it represents all the positive integers not ruled out for representation by congruence conditions. In more modern terminology, the regular forms are those for which a local-global principle holds for the representation of integers. Since that time, quadratic forms and lattices with this property and various natural generalizations of it have been studied extensively, both over the ring of rational integers and over Hasse domains in general global fields. In this talk, we will give an overview of some of the main results that have been obtained, describe some recent results, and indicate some remaining open problems on these interesting classes of lattices.

Lattices which are spinor regular but not regular

Anna Haensch

A lattice is called (spinor) regular if it represents everything that is represented by its (spinor) genus. In 1997, Jagy, Kaplansky and Schiemann proved that there at most 913 lattices which are regular, and proved that all but 22 of the 913 are indeed regular. Of the remaining lattices, 8 were later confirmed as regular by Oh and the final 14 by Lemke Oliver. Later, Jagy produced a list of spinor regular lattices which are not regular up to discriminant 575,000, conjecturing it to be complete. In this talk I will discuss a recent joint project with Andrew Earnest in which we consider the completeness of Jagy's list.

Weyl Group Multiple Dirichlet Series and a Metaplectic Eisenstein Series on GL(3,R).

Edmund Karasiewicz

The theory of Weyl group multiple Dirichlet series (Dirichlet series in several variables with a group of functional equations isomorphic to a Weyl group) has been developed by Brubaker, Bump, Chinta, et al. These multiple Dirichlet series are conjectured to be the Fourier coefficients of metaplectic Eisenstein series. The work of Brubaker, Bump, Chinta, et al. includes a hypothesis that forces the base field to be totally imaginary. We will consider a specific metaplectic Eisenstein series over R and see how this fits into the existing theory.

Algorithms for lattices

Markus Kirschmer

We will discuss several algorithms for lattices in definite orthogonal or hermitian spaces. For example how to find isometries or how to construct a lattice in a genus given only by local invariants. These algorithms have recently be used to classify all quadratic and hermitian lattices with class number at most 2.

Self-Adjoint Operators and Zeros of L-functions

Kim Klinger-Logan

Hilbert and Polya raised the possibility of proving the Riemann Hypothesis by producing self-adjoint operators with eigenvalues $s(s-1)$ for zeros s of $\zeta(s)$. A spark of hope appeared when Haas (1977) numerically miscalculated eigenvalues for the automorphic Laplacian and obtained zeros of zeta in his list of s-values. In 1981, Hejhal identified the flaw and determined what Haas had actually computed – not genuine eigenvalues but parameters in an automorphic Helmholtz equation (the time-independent, stationary version of the wave equation). ColinDeVerdière speculated on a possible legitimization that languished for 30 years. Recent work of Bombieri and Garrett makes precise ColinDeVerdière's speculations and proves that, while the discrete spectrum (if any) must have s-values among zeros of corresponding zeta functions (expressed as periods of Eisenstein series), there is an operator-theoretic mechanism by which the regular behavior of zeta on the edge of the critical strip coerces discrete spectrum to be too regularly spaced to be compatible with Montgomery's pair-correlation conjecture. Similar mechanisms are shown to apply to more complicated non-compact periods of Eisenstein series producing L-functions. These operator-theoretic mechanisms also open up further possibilities bearing on the locations of zeros of L-functions.

Representations of integral Hermitian forms by sums of norms

Jingbo Liu

In 1770, Lagrange proved the famous four square theorem, which says that each positive integer a can be represented as a sum of four squares. This theorem has been generalized in many directions since then. One interesting generalization is to consider the representation of integral quadratic forms of more variables by sums of squares.

Let $g_{\mathbb{Z}}(n)$ be the smallest number of squares whose sum represents all positive definite integral quadratic forms in n variables over $\mathbb{Z}$ that are represented by some sums of squares. In 1996, Icaza first proved the existence of $g_{\mathbb{Z}}(n)$ and she also gave an explicit upper bound for it. An improved upper bound was obtained later by Kim and Oh in 2005.

Similarly, for Hermitian forms over the ring of integers $\mathcal{O}_E$ of imaginary quadratic field E, we define $g_E(n)$ to be the smallest number of norms whose sum represents all positive definite integral Hermitian forms of n variables over $\mathcal{O}_E$ that are represented by some sums of norms. In this talk, we will present a generalization of Kim and Oh's method and give an explicit upper bound for $g_E(n)$ for any imaginary quadratic field E and positive integer n.

On Strictly k-Regular Quadratic Forms

Alicia Marino

An integral quadratic form is said to be strictly k-regular if it primitively represents all quadratic forms of k variables that are primitively represented by its genus. We show that, for $k > 1$, there are finitely many inequivalent positive definite primitive integral quadratic forms of $k+4$ variables that are strictly k-regular. Our result extends a recent finiteness result of Andrew Earnest et al. (2014) on strictly regular quadratic forms of 4 variables.

Gaps Between Zeros of $GL(2)$ L-functions

Steven J Miller

This talk is a report on recent progress made with REU students on gaps between zeros of L(s,f) for a primitive holomorphic cusp form f on GL(2). Combining mean value estimates from Montgomery and Vaughan with a method of Ramachandra, we prove a formula for the mixed second moment of derivatives of L(1/2+it,f) and use it to show that there are infinitely many gaps between consecutive zeros of L(s,f) along the critical line that are at least sqrt(3) times the average spacing.

This work is joint with Owen Barrett, Brian McDonald, Patrick Ryan, Caroline Turnage-Butterbaugh and Karl Winsor.

Elliptic Curves with Maximally Disjoint Division Fields

James Ricci

One of the many interesting algebraic objects associated to a given elliptic curve defined over the rational numbers, $E/\mathbf{Q}$, is its full-torsion representation $\rho_E : \mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q}) \to \mathrm{GL}_2(\hat{\mathbf{Z}})$. Generalizing this idea, one can create another full-torsion Galois representation, $\rho_{(E_1,E_2)} : \mathrm{Gal}(\bar{\mathbf{Q}}/\mathbf{Q}) \to \left(\mathrm{GL}_2(\hat{\mathbf{Z}})\right)^2$ associated to a pair (E_1, E_2) of elliptic curves defined over $\mathbf{Q}$. The goal of this talk is to provide an infinite number of concrete examples of pairs of elliptic curves whose associated full-torsion Galois representation $\rho_{(E_1,E_2)}$ has maximal image.

The Shintani Lift on the Full Modular Group

Karen Taylor

We will describe the Shintani lift (of cusp forms on the full modular group), its proof , connections to cycle integrals and a possible generalization formed from the nth hyperbolic fourier coefficients of cusp forms.

On the field of definition of a cubic rational function and its critical points

Bianca Thompson

Using essentially only algebra, we give a proof that a cubic rational function over $\mathbb{C}$ with only real critical points is equivalent to a real rational function. We also determine all fields $\mathbb{Q}_p$ over which a reasonable generalization holds.

Quadratic forms and orthogonal modular forms

John Voight

We give an overview of the theory of algebraic modular forms in the special case of a definite orthogonal group. We describe Hecke operators in the language of quadratic forms, exhibit algorithms for computing them, and present some recent results characterizing orthogonal modular forms in low rank.

Representation numbers and the Eichler Commutation Relation

Lynne Walling

Given a positive definite quadratic form Q on a lattice L, the degree n Siegel theta series attached to L is a modular form whose Fourier coefficients tell us how often L represents any given n-dimensional quadratic form. We look at the action of the "Hecke operators" on these modular forms to get relations on the average representation numbers, where the average is over the genus of L. (We observe that the average representation numbers for the genus of L are asymptotic to the representation numbers of L.) We briefly discuss the obstacles in using modular forms to give us closed-form formulas for these average representation numbers.

A Chinese remainder theorem for Galois representations

Siman Wong

For any list of not necessarily distinct quartic polynomials $g_1, ..., g_n \pmod 2$ with non-zero constant terms and any list of distinct primes $q_1, ..., q_n$ all greater than 53, we construct infinitely many surjective Galois representations

$$\rho : G_Q \to GL_4(F_2)$$

where the characteristic polynomial of ρ at $Frob(q_i)$ is equal to g_i for all i. Using the effective Chebotarev density theorem for function fields and geometric Galois extensions of $\mathbb{Q}(t)$ with $GL_d(F_q)$-Galois groups, we reduce this to a problem about geometric algebra and permutation groups; the exceptional isomorphisms

$$A_8 \sim GL_4(F_2) \sim P\Omega_6^+(F_2)$$

play a key role. We also have similar results for other residual Representations.

Computation of certain modular forms using polytopes

Dan Yasaki

The cohomology of arithmetic groups is built from certain automorphic forms, allowing for explicit computation of Hecke eigenvalues using topological techniques in certain cases. For modular forms attached to the general linear group over a number field F of class number one, these cohomological forms can be described in terms an associated Voronoi tessellation coming from the study of perfect n-ary forms over F. In this talk we describe this relationship and give several examples of these computations.