

PROBLEM 6

Let p be a prime. Determine the number of homomorphisms from $\mathbb{Z}_p \oplus \mathbb{Z}_p$ into \mathbb{Z}_p .

SOLUTION TO PROBLEM 6

To answer this question, first let's consider how many homomorphisms there are from \mathbb{Z}_p into itself. Since p is prime, a homomorphism can send 1 to any element of p , therefore there are a total of p homomorphisms. Now any homomorphism

$$\Phi : \mathbb{Z}_p \oplus \mathbb{Z}_p \rightarrow \mathbb{Z}_p$$

is completely determined by where it sends $(1, 0)$ and $(0, 1)$, therefore there are p^2 possibilities.