

PROBLEM 5

Suppose that ϕ is a homomorphism of $\mathcal{U}(30)$, $\text{Ker}(\phi) = \{1, 11\}$ and $\phi(7) = 7$. Determine all elements that map to 7.

SOLUTION TO PROBLEM 5

Recall that

$$\mathcal{U}(36) = \{1, 7, 11, 13, 17, 19, 23, 29\},$$

and since $\text{Ker}(\Phi) = \{1, 11\}$ we know that Φ is a 2-to-1 map. This means there are precisely two elements that map to 7, and one of them is 7. Based on this, we know that

$$\Phi^{-1}(7) = 7\text{Ker}(\Phi),$$

and hence $\Phi^{-1}(7) = \{7, 7 \cdot 11\} = \{7, 17\}$.