

PROBLEM 4

Give an example of a subset of a ring that is a subgroup under addition, but not a subring.

SOLUTION TO PROBLEM 4

The subgroup of the rationals given by

$$\left\{ \frac{a}{2} : a \in \mathbb{Z} \right\} \subseteq \mathbb{Q}.$$

Then for any $a, b \in \mathbb{Z}$,

$$\frac{a}{2} - \frac{b}{2} = \frac{a-b}{2}$$

but

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

The subgroup of \mathbb{R} given by $\{n\sqrt{2} : n \in \mathbb{Z}\}$ would also work.