

PROBLEM 2

Suppose a belongs to a ring and $a^4 = a^2$. Prove that $a^{2^n} = a^2$ for all $n \geq 1$.

SOLUTION TO PROBLEM 2

Suppose a belongs to a ring and $a^4 = a^2$. We will prove that $a^{2^n} = a^2$ for all $n \geq 1$ using the principle of induction. For the base case, we see that when $n = 1$, clearly $a^2 = a^2$. Suppose that $a^{2^n} = a^2$. Then

$$a^{2^{(n+1)}} = a^{2^n} a^2 = a^2 a^2 = a^4 = a^2,$$

and we are done.