Intro to Groups	Groups	Finite Groups; Subgroups	Cyclic Groups	Permutation Groups
$200	$200	$200	$200	$200
$400	$400	$400	$400	$400
$600	$600	$600	$600	$600
$800	$800	$800	$800	$800

What is the symmetry group of this object?

Answer

Cyclic rotation group of order 3.

What group theoretic property do the uppercase letters F, G, J, L, P, Q and R have that is not shared by the remaining uppercase letters of the alphabet?

Answer

Their only symmetry is the identity.

Give an example that proves the dihedral group is non abelian.

Answer

DR_1=H but R_1D=V

Give an example of an object whose symmetry group is D_5.

Answer

What does it mean for a binary operation to be closed? Given an example of two binary operations on Z, one closed, one not closed.

Answer

A binary operation on Z is closed if it maps into Z. Addition on Z is closed, but division is not.

What is U(15)? List all elements.

Answer

$U(15)=\{1,2,4,7,8,11,13,14\}$

This is Groups2 for $600

Answer

This is the answer.

Prove that every group table is a Latin Square. That is, every element in the group appears exactly once in each row and each column.

Answer

It can’t show up more than once, by the cancelation property; that is, if

$ab=c$ and $ad=c$

then $b=d$.

It must show up at least once, since every element has an inverse, and

$ab=e$

implies

$a(bc)=c$.

Define Z(G).

Answer

$Z(G)=\{a \text{ in } G : ag=ga \text{ for all } g \text{ in } G\}$

Recall, C(a)={ g in G: ga=ag} is the centralizer. Must the centralizer be abelian? Why or why not?

Answer

No, for example, in D_4 we have C(R_2)=D_4.

D_4 has 7 cyclic subgroups, list them.

Answer

$D_4=\langle R_1\rangle=\langle R_3\rangle$

$\{1\}$

$\langle R_2\rangle$

$\langle V\rangle$

$\langle H\rangle$

$\langle D\rangle$

$\langle D'\rangle$

Recall, G=GL(2,R) is the group of 2x2 matrices with non-zero determinant, under mult.

Prove that
H={A in GL(2,R): det(A) is a power of 2}
is a subgroup of G.

Answer

Use one-step subgroup test. Assume A,B in H. Then det(A)*det(B^-1)=det(A)/det(B).

Define <a>

Answer

<a>={a^n : n is in Z}

How many subgroups of order 3 does Z_9 have?

Answer

1

Determine the subgroup lattice for $Z_{(p^2q)}$ where p and q are distinct primes.

Answer

Look over there →

Give an example of an infinite group that has exactly two elements of order 4.

Answer

The complex numbers. Both i and -i have order 4.

Define S_n. What are its elements, what is its binary operation.

Answer

S_n is the group of permutations of n elements. The group elements are permutaitons. The group operation is composition of functions.

What is the order of (345)(245)

Answer

2

How many elements of order 5 are there in S_7?

Answer

There are

(7*6*5*4*3)/5=504

Prove that (1234) is not the product of 3-cycles.

Answer

$(1234)=(14)(13)(12)$

but any product of three cycles has even order, since

$(abc) =(ac)(ab)$