Homework 9

1. Let (a, b) belong to $\mathbb{Z}_m \oplus \mathbb{Z}_n$. Prove that $\mid (a, b) \mid$ divides $\text{lcm}(m, n)$.
2. Express $U(165)$ as an external direct product of cyclic groups of the form $\mathbb{Z}_n$.
3. Find an integer n so that $U(n)$ contains a subgroup isomorphic to $\mathbb{Z}_5 \oplus \mathbb{Z}_5$.
4. If N and M are normal subgroups of G, prove that NM is also a normal subgroup of G.
5. Suppose that N is a normal subgroup of G and $\mid G/N \mid = m$. Prove that $x^m \in N$ for every $x \in G$.