Homework 7

1. Let G be a group and let $g \in G$. If $z \in Z(G)$, show that the inner automorphism induced by g is the same as the inner automorphism induced by zg (that is, the mapping ϕ_g and ϕ_{zg} are equal).

2. Suppose that g and h induce the same inner automorphism of a group G. Prove that $h^{-1}g \in \mathbb{Z}(G)$.

3. Combine the results of exercises 1 and 2 into a single "if and only if" statement.

4. Let $H = \{0, \pm 3, \pm 6, \pm 9, ...\}$. Find all the left cosets of H in $\mathbb{Z}$.

5. Suppose that a has order 15. Find all of the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$.

6. If H and K are subgroups of G and $g \in G$, show that $g(H \cap K) = gH \cap gK$.