Homework 5

1. Determine whether the following permutations are even or odd

   (a) (135)

   (b) (12)(134)(152)

   (c) (13567)

2. Prove that A_n is non-abelian for any $n \geq 4$. [*Hint: You can show that S_n is not abelian for $n \geq 3$ by observing that* $(123)(12) \neq (12)(123)$.]

3. How many elements of order 5 are there in A_6?

4. If α and β are distinct 2-cycles, what are the possibilities for $\mid \alpha\beta \mid$?

5. Suppose $\alpha = (a_1 a_2 ... a_n)$ is an n-cycle. Prove that α has order n. That is, show that for any $1 \leq i \leq n$,

   $$\alpha^n(a_i) = a_i,$$

   and n is minimal positive integer for which this holds. It will be helpful to note that for any a_i, we have

   $$\alpha^k(a_i) = a_{i+k \mod n}.$$