Homework 4

1. Determine the subgroup lattice for $\mathbb{Z}_8$.

2. Let G be a group and let $a \in G$. Prove that $\langle a \rangle = \langle a^{-1} \rangle$.

3. Prove that $\mathbb{Z}_n$ has an even number of generators if $n > 2$. What does this tell you about $\phi(n)$?

4. Write the following symmetries as a product of 2-cycles.

   (a) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 1 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 5 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{bmatrix}$

5. What is the order of a product of a pair of disjoint cycles of order 4 and 6?

6. Prove that A_n is non-abelian for any $n \geq 4$. [*Hint: You can show that S_n is not abelian for $n \geq 3$ by observing that* $(123)(12) \neq (12)(123)$]