

HOMEWORK 3

1. Prove that in a group $(a^{-1})^{-1} = a$.
2. Give an example of a group with 105 elements. Give two examples of groups with 44 elements.
3. Prove that a group of even order must have an element of order 2.
4. Let m and n be elements of the additive group \mathbb{Z} .
 - (a) Prove that $\langle m \rangle \cap \langle n \rangle$ is a group.
 - (b) Find a generator for the group $\langle m \rangle \cap \langle n \rangle$.
5. Prove that a group of order 3 must be cyclic.
6. Suppose that a and b belong to a group G , a and b commute, and $|a|$ and $|b|$ are finite. Prove that G has an element of order $\text{lcm}(|a|, |b|)$. [Recall: $\text{lcm}(m, n)$ denotes the least common multiple of any two numbers m and n .]