Homework 2

1. Give two reasons why the set of odd integers under addition is not a group.

2. Prove that the set of all 2×2 matrices with entries from $\mathbb{R}$ and determinant $+1$ is a group under matrix multiplication.

3. Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$.

4. Prove that in any group, an element and its inverse have the same order.

5. Prove that if an abelian group has more than three elements of order 2, then it has at least 7 elements of order 2. Find an example that shows this is not true for non-abelian groups.

6. Let G be a group. Show that $Z(G) = \cap_{a \in G} C(a)$. [This means the intersection of *all* subgroups of the form $C(a)$.]