

HOMEWORK 11

1. let \mathbb{R} be the group of real numbers under addition. Prove that the map from \mathbb{R} to $GL(2, \mathbb{R})$ that sends x to

$$\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

is a homomorphism. What is the kernel of the homomorphism? *Hint: You may need some properties of sine and cosine, check the back cover of your calculus textbook.*

2. If H and K are normal subgroups of G and $H \cap K = \{e\}$, prove that G is isomorphic to a subgroup of $G/H \oplus G/K$.
3. What is the smallest positive integer n such that there are exactly 4 nonisomorphic abelian groups of order n ? Name the four groups.
4. The set $\{1, 9, 16, 22, 29, 53, 74, 79, 81\}$ is a group under multiplication modulo 91. Determine the isomorphism class of this group.
5. Verify the corollary to the Fundamental Theorem of Finite Abelian Groups in the case that the group has order 1080 and the divisor is 180. *Hint: See the example immediately following the corollary on page 231.*