Homework 10

1. Let $\mathbb{R}^*$ be the group of nonzero real numbers under multiplication, and let r be a positive integer. Show that he mapping that takes x to x^r is a homomorphism from $\mathbb{R}^*$ to $\mathbb{R}^*$ and determine the kernel. Which values of r yield an isomorphism.

2. Suppose that $\phi : \mathbb{Z}_{50} \to \mathbb{Z}_{15}$ is a group homomorphism with $\phi(7) = 6$.

   (a) Determine $\phi(x)$.

   (b) Determine the image of ϕ.

   (c) Determine the kernel of ϕ.

   (d) Determine $\phi^{-1}(3)$.

3. Explain why the correspondence $x \mapsto 3x$ from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{10}$ is not a homomorphism.

4. Prove that there is no homomorphism from $\mathbb{Z}_{16} \oplus \mathbb{Z}_2$ to $\mathbb{Z}_4 \oplus \mathbb{Z}_4$.

5. Suppose that G is a group, and suppose that for every prime p there is a homomorphism ϕ_p so that the image of G under ϕ_p is $\mathbb{Z}_p$. What can you say about G? Give an example of a group that satisfies this property.