1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, ...

..., 300, 301, 302, 303, 304, 305, 306,307, 308, 309, 310, 311, 312,313, 314, 315, 316,..

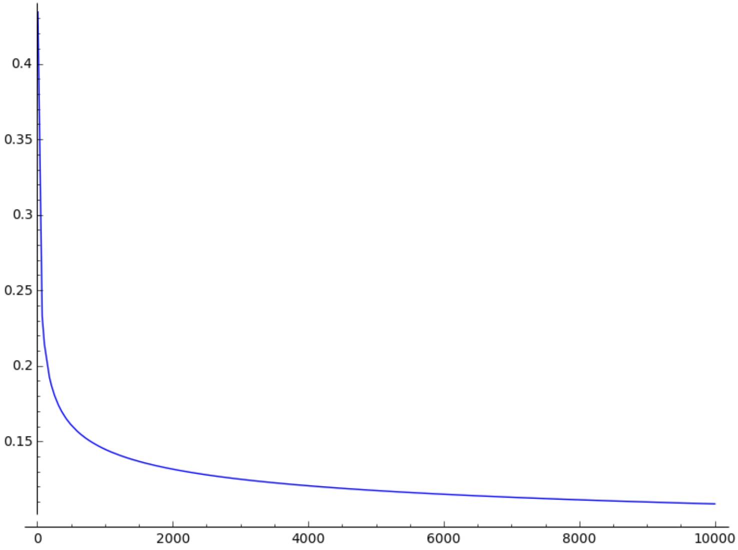
..., 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038,...

Definition: For a natural number n, let $\pi(n)$ denote the number of primes less than n.

n	$\pi(n)$	$\frac{\pi(n)}{n}$	$\frac{1}{\ln(n)}$	$\frac{\frac{\pi(n)}{n}}{\frac{1}{\ln(n)}}$
5	2	.4	.62	0.64516...
10	4	.4	.43429...	0.92104...
100	25	.25	.21714...	1.15133...
1,000	168	.168	.14476...	1.16054...
10,000	1229	.1229	.10857...	1.13199...
100,000	8592	.09592	.08685...	1.10443...
1,000,000	78498	.078498	.07238...	1.08452...

The Prime Number Theorem: As n approaches infinity, the proportion of primes less than or equal to n approaches $\frac{1}{\ln(n)}$, Specifically,

$$\lim_{n \to \infty} \left(\frac{\pi(n)/n}{1/\ln(n)} \right) = 1.$$



$$f(x) = \frac{1}{\ln(x)} \sim \frac{\pi(x)}{x}$$

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, ...

..., 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312,313, 314, 315, 316,..

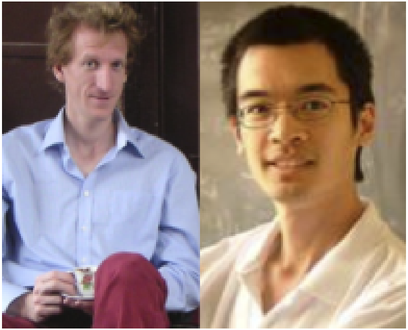
..., 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038,...

Twin Prime Conjecture: There are infinitely many primes whose difference is 2.

Yitang Zhang, January 2013

James Maynard, November 2013

Green and Tao's Theorem: There are arbitrarily long arithmetic progressions of primes.



Ben Green and Terrence Tao, 2004

Question: Is there some sort of generating function for primes?

Marin Mersenne, 1588-1648

Frank Nelson Cole, 1861-1926

Definition: A *Mersenne Prime* is a prime of the form $2^p - 1$, where p is prime. A prime of the form $2^{2^k} + 1$ is called a *Fermat Prime*.