
HOMEWORK #2

1. For the Diophantine equation $123x + 360y = 99$ determine
 - (a) all its solutions;
 - (b) all its solutions in positive integers.
2. At the fruit counter in a store, apples are sold 5 cents each and oranges are sold 7 cents each. Say Peter purchases 4 apples and 12 oranges. Peter notices that Grace also bought apples and oranges and she pays the same total amount as he did, but with a different number of apples and oranges. Knowing that Grace purchased at least 3 oranges, does Peter have enough information to determine the exact number of apples and oranges purchased by Grace?
3. When Mr. Smith cashed a check at his bank, the teller mistook the number of cents for the number of dollars and vice versa. Unaware of this, Mr. Smith spent 68 cents and then noticed to his surprise that he had twice the amount of the original check. Determine the smallest value for which the check could have been written. [Hint: You may assume that the amount of the original check was less than \$100, or else the transposition doesn't make sense.]
4. The greatest common divisor of nonzero integers a_1, \dots, a_n , denoted $\gcd(a_1, \dots, a_n)$, is defined to be the largest positive common divisor of a_1, \dots, a_n .
 - (a) Show that $\gcd(a_1, \dots, a_n) = \gcd(a_1, \gcd(a_2, \dots, a_n))$ when $n > 2$
 - (b) Part (a) indicates that we can find the gcd of three or more integers by performing the Euclidean algorithm more than once. Do that for $\gcd(18777, 1848, 3960)$.
 - (c) Use part (a) to show that $\gcd(a_1, \dots, a_n)$ is of the form $a_1x_1 + a_2x_2 + \dots + a_nx_n$, where x_1, \dots, x_n are integers. [Hint: induction.]