Section 2.7 Problem Set 1

7.1 For the following, if the statement is true, explain why. If it is false, give a counterexample.

(a) The rank of a matrix is equal to the number of its non-zero columns.

(b) The $m \times n$ zero matrix is the only $m \times n$ matrix having rank 0.

(c) Elementary row operations preserve rank.

(d) Elementary column operations do not necessarily preserve rank.

(e) The rank of a matrix is equal to the maximum number of linearly independent columns in the matrix.

(f) The rank of a matrix is equal to the maximum number of linearly independent rows in the matrix.

(g) The rank of an $n \times n$ matrix is at most n.

(h) An $n \times n$ matrix having rank n is invertible.

7.3 Compute rank and find bases for $\ker(A), \text{range}(A), \ker(A^T)$ and $\text{range}(A^T)$ for

(a) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$