Section 2.2 Solutions

2.1 a) $$\begin{bmatrix}1 & 2 & -1\\ 2 & 2 & 1\\ 3 & 5 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}-1\\ 1\\ -1\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}4\\ -3\\ -1\end{bmatrix}$$

b) $$\begin{bmatrix}1 & -2 & -1\\ 2 & -3 & 1\\ 3 & -5 & 0\\ 1 & 0 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}1\\ 6\\ 7\\ 9\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}9\\ 4\\ 0\end{bmatrix} + x_3\begin{bmatrix}-5\\ -3\\ 1\end{bmatrix}$$

c) $$\begin{bmatrix}1 & 2 & 0 & 2\\ 3 & 5 & -1 & 6\\ 2 & 4 & 1 & 2\\ 2 & 0 & -7 & 11\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}6\\ 17\\ 12\\ 7\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}2\\ 3\\ -2\\ -1\end{bmatrix}$$

d) $$\begin{bmatrix}1 & -4 & -1 & 1\\ 2 & -8 & 1 & -4\\ -1 & 4 & -2 & 5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}3\\ 9\\ -6\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}\frac{21}{5}\\ \frac{-3}{20}\\ \frac{3}{5}\\ 0\end{bmatrix} + x_4\begin{bmatrix}\frac{7}{5}\\ \frac{-3}{10}\\ \frac{6}{5}\\ 1\end{bmatrix}$$

e) $$\begin{bmatrix}1 & 2 & -1 & 3\\ 2 & 4 & -1 & 6\\ 0 & 1 & 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}2\\ 5\\ 3\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}-3\\ 3\\ 1\\ 0\end{bmatrix} + x_4\begin{bmatrix}13\\ -2\\ 12\\ 1\end{bmatrix}$$

f) $$\begin{bmatrix}2 & -2 & -1 & 6 & -2\\ 1 & -1 & 1 & 2 & -1\\ 4 & -4 & 5 & 7 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 6\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}-23\\ 0\\ 7\\ 9\\ 0\end{bmatrix} + x_2\begin{bmatrix}1\\ 1\\ 0\\ 0\\ 0\end{bmatrix} + x_5\begin{bmatrix}-23\\ 0\\ 6\\ 9\\ 1\end{bmatrix}$$

g) $$\begin{bmatrix}3 & -1 & 1 & -1 & 2\\ 1 & -1 & -1 & -2 & -1\\ 5 & -2 & 1 & -3 & 3\\ 2 & -1 & 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\end{bmatrix} = \begin{bmatrix}5\\ 2\\ 10\\ 5\end{bmatrix}, \text{ solution } \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} = \begin{bmatrix}3\\ 7\\ 0\\ -3\\ 0\end{bmatrix} + x_3\begin{bmatrix}-1\\ -2\\ 1\\ 0\\ 0\end{bmatrix} + x_5\begin{bmatrix}-2\\ -5\\ 0\\ 1\\ 1\end{bmatrix}$$

2.2 To find all solutions to the vector equation

$$x_1v_1 + x_2v_2 + x_3v_3 = \vec{0}$$

where

$$v_1\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}, v_2\begin{bmatrix}0\\ 1\\ 1\end{bmatrix}, \text{ and } v_3\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

we are essentially finding solutions to the system of equations

$$\begin{aligned} x_1 + x_3 &= 0\\ x_1 + x_2 &= 0\\ x_2 + x_3 &= 0. \end{aligned}$$

Therefore we will set up the augmented matrix,

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right].$$

Putting this matrix in echelon form, we obtain

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right],$$

and therefore we may conclude that $x_1 = x_2 = x_3 = 0$. Hence the system is linearly independent, since only the trivial linear combination yields the zero vector.