Theorem. Let A and B be subsets of a universal set U. If $A \subseteq B$, then $B^c \subseteq A^c$.

Proof. Let A and B be subsets of a universal set U. We assume that $A \subseteq B$, and we will show directly that $B^c \subseteq A^c$. Since $A \subseteq B$, we know that for every $x \in U$, if $x \in A$, then $x \in B$. Because we want to show that $B^c \subseteq A^c$, we must establish that for every $y \in U$, if $y \in B^c$, then $y \in A^c$. Hence, we choose $y \in B^c$. Since $y \in B^c$, we know that $y \notin B$. By the contrapositive of the conditional statement "if $x \in A$, then $x \in B$" (which we have assumed is true), we know that it is also true that for all $x \in U$, if $x \notin B$, then $x \notin A$. Hence, since we have established that the chosen element y is not in the set B, it follows that y is not in the set A. In other words, $y \in A^c$.

Having proven that for all $y \in U$, if $y \in B^c$, then $y \in A^c$, it follows by definition that $B^c \subseteq A^c$, and this proves the theorem. □