Proof Portfolio Guidelines

Project Description

During the course of this semester, you will be learning to construct and write mathematically rigorous arguments. This will involve learning some formal elementary logic, being introduced to some of the more common types of proof structures, and developing your mathematical writing style. We will work on these writing skills through our encounters with a multitude of mathematical topics, some of which may be familiar to you. You will work throughout the semester to build this Proof Portfolio, which is designed to showcase your growing expertise in formal mathematical communication and proof-writing.

Problems

Throughout the semester you will be given a list of problems (among which you have some choice); each asks you to prove or disprove a mathematical claim, possibly along with some additional directions. Throughout the term, you will essentially be working on two portfolio problems per week, and you will be supported in maintaining this goal through certain weekly deadlines. Ultimately, your portfolio will consist of 10 completed problems.

Drafts

Part of your grade on this portfolio is based on submission of preliminary drafts for me to critique. Once you have a draft of a proof (that is, a typeset, formal version of what you believe to be a good proof, not a sketch of ideas or scratch work), you will submit it to me electronically through overleaf.com. I will make comments on your work and assign a provisional grade using the scheme explained in this document; the file will be returned to you by email. You should then do any necessary revisions before resubmitting it. Each problem is to be submitted twice in draft form before a final grade is awarded.

Almost every week of the semester, you will be required to submit a draft of one new problem and a revised draft of one earlier problem. By doing this work over 14 weeks' time, you will progress consistently through the required 10 problems and find that the project is on track for completion and final submission at the end of the course.

The drafts are given provisional grades that help you understand the current state of your work. The third draft of any portfolio problem is final; your third draft will be submitted in hard copy format in your final portfolio on the last day of class, **Thursday, April 21st**. *You are welcome – indeed, encouraged – to discuss your ideas in person with me on any given problem at any stage: before or after submitting either draft 1 or draft 2.*

Deadlines

Starting on **Friday January 15th** a first draft of your first problem is due by 5pm. From this point onwards you are expected to submit two problems each week: one on Tuesday before 5pm and one on Friday before 5pm. Each week you should be submitting one first draft and one second draft, but it is up to you the order in which you submit the problems. The last date you will be able to submit a draft is Friday April 15th.

- There will be no drafts due on the Fridays after Exams 1 and 2.
- There will be no drafts due during the week of spring break.
- Because of Easter Break, there will be no drafts due during the week of March 21-25.

Rules, Structure, and Expectations for the Project

1. This project comes with high expectations for academic honesty. In particular, the portfolio is an *independent* project that you should view as a semester-long take home examination. You may not discuss the problems with anyone except your current 250 instructor, Anna Haensch. This means you cannot talk to other students about your solutions (or even which problems you are choosing to work on), nor can you ask for help from the Tutoring Center tutors or other professors, nor from any person in an online setting. Violation of this policy is grounds for failure of the course.

   The principal reason for this requirement of independence is that when you finish Math 250, *you* need to be competent and comfortable with proving mathematical statements so that you can be successful in future classes. These 10 problems are designed to help you achieve that level of competence.

2. You may not refer to any sources other than the textbook for this course. This includes a prohibition against searching for solutions or ideas on the Internet unless you are directed specifically to do so by your instructor in the context of a problem. If you think that you need some background material or a definition from another source then you may ask me for permission, and if granted then you may look up the necessary material and include it with a footnote in your proof. Using solutions found on the internet to portfolio problems is grounds for failure of the course.

3. All drafts, both preliminary and final, must be typeset and follow the guidelines for mathematical writing that are in our textbook and on the handout that is available on the course web page. Guidance on "typesetting" is provided later in this document.

4. Each of the ten problems is individually worth 10 points. In addition, there are 20 required drafts (2 for each of the 10 problems), and you will receive 20/20 for meeting all 20 of the draft deadlines. For each deadline you miss, there is a 2-point deduction. Note well: if you fall behind, you can only ever submit 2 drafts in a given week, so you begin forfeiting opportunities for review by missing deadlines.

5. The final version of proof portfolio (in printed, hard-copy form) is due on the last day of class, Thursday, April 21st. Only the scores on the final versions will count toward your grade.

Grading

The portfolio will be graded on a scale of 120 points. This 120 point grade counts 20% of your grade in the course.

Each of the 10 problems will be worth 10 points (for a total of 100 points), and in addition, there will be 20 points possible for meeting the aforementioned draft deadlines. For the problems themselves, grades will be awarded according to the following scheme:

10 points	Perfect. Valid reasoning, follows all guidelines for writing, no errors of any kind.
9 points	Valid reasoning, follows all guidelines for writing, but minor error (note singular) in writing.
7 points	Significant mathematical progress has been made towards a proof but either the argument has one major error or it is does not yet meet the writing guidelines.
5 points	Some significant mathematical progress has been made towards a valid proof but there are key errors present in the mathematics or major issues with the written presentation.
3 points	Evidence of having at least one good idea and making an effort to write a formal proof.
0 points	Essentially no progress has been made towards a valid proof.

Both grades on provisional drafts and final versions of each problem will be given according to this scale. Grades on provisional drafts are given to help you see where your progress on the problem stands. Only grades on final drafts count towards the 120 point total. Half or single points may be awarded ($\pm$) at the instructor's discretion.

Electronic Submission of Portfolio Problems

Each solution or proof must be typeset and presented in .pdf format.

To this end, you are required to learn to use LaTeX (pronounced "lay-TECH" or "lah-TECH", but never like "latex gloves"), the professional typesetting software of choice for mathematicians (and in which this document is written). While it takes some modest effort at first to learn this program, it is by far the most superior option and will be useful throughout your work in future mathematics courses (and others, likely).

Rather than taking the time and trouble to install LaTeXon our own computers, we will use an online interface called Overleaf.

Prof. Robert Talbert from Grand Valley State University has developed a sequence of screencasts that explain what LaTeX is and how to get started using it. See the following video from the GVSU Math YouTube channel at

http://gvsu.edu/s/rg

and watch "What is LaTeX?" From there, you can explore "Your first LaTeX document," "Basic mathematics in LaTeX," and so on. I am willing and able to offer support to you in learning this software; we will spend some classtime on it working to help everyone get up to speed.

Each draft will be submitted to me electronically by sending me a link via email at `haenscha@duq.edu` to your overleaf page. My email will auto sort your submissions, so your subject line should be as follows (except replace relevant problem/draft numbers as necessary):

`subject`: Math 250 Problem 3B Draft 2

I will mark up the file (either the .pdf or .tex) with my comments and send it back to you.

Frequently Asked Questions

Following are some (asked and anticipated) questions about this Portfolio. The answers to these questions contain some important additional guidelines and expectations for the Portfolio Project. You should read all of this carefully and follow the instructions accordingly.

What other requirements are there for my Portfolio Proofs?

The solution for each problem must be written using complete sentences and according to the writing guidelines specified in the text. Proper grammar, proper sentence and paragraph structure, and correct spelling are necessities. Papers that do not adhere to these basic requirements will receive a draft score of zero.

What happens if I submit an incorrect or incomplete solution?

Each time you submit a draft, I will return your problem and indicate if it is finished and ready for inclusion in the final portfolio or if it needs more work. When you submit a draft for review, you are asking me, "Is this good enough for my Portfolio?" I will respond with comments and suggestions, as well as a provisional grade, in order to help you to assess your progress. While provisional grades are given as a guide (and are part of receiving 20 points of credits for drafts), there is no penalty for a grade lower than 10 *until the third and final draft is submitted*.

Can I work with someone else or use sources other than the textbook?

No. No collaboration is permitted. The only person you can discuss these problems with is your instructor, Anna Haensch, and the only resource you may use is the textbook. Use of other human or intellectual resources is considered plagiarism and is not acceptable. One of the primary goals of Math 250 is that you acquire deep personal understanding of proof techniques and the ability to read and write proofs. Being able to do so independently is essential, and thus this project is an independent endeavor.

Can I come to your office for help?

Most certainly. You are welcome at any time during office hours or when my door is open to discuss questions on the portfolio (or any other aspect of the course). If my stated times do not suit your schedule, please request an appointment (ideally, at least 24 hours in advance). There is only one requirement for you when you come to seek help: *do not come empty-handed*. By this I mean that you should not come unprepared saying that you "have no idea where to start." Part of learning to write proofs is thinking of possible ways to start, even if those ways turn out to be wrong. When you have chosen a problem to work on, start your scratch work with a list of things that you know which seem like they might be related. Write down what you know and what you need to show, and see if any of your ideas help with even a small part of this task.

What criteria will be used to judge my proofs?

A proof must be logically correct and mathematically valid. In addition, it must be written according to the stated guidelines distributed in class (that will also be available from the course web page, or in more complete form in Appendix A of the text). You will get a better feel for how the grading process works by simply submitting drafts for review.

Below follows a model solution to a sample Portfolio Problem.

Conjecture X. If x and y are real numbers, then $\dfrac{x+y}{2} \geq \sqrt{xy}$.

This proposition is false as is shown by the following counterexample[1]: Letting $x = -2$ and $y = -2$, observe that it follows that

$$\frac{x+y}{2} = \frac{-2-2}{2} = -2, \text{ but } \sqrt{xy} = \sqrt{(-2)(-2)} = 2.$$

In this case, $\frac{x+y}{2} < \sqrt{xy}$. This shows that the given proposition is false, because our example demonstrates that the hypothesis of the conditional statement can be true, but the conclusion of the conditional statement false. □

However, based on a large collection of examples where x and y are both positive, it appears that if x and y are nonnegative real numbers, then $\frac{x+y}{2} \geq \sqrt{xy}$. We will state this as a theorem and prove it.

Theorem: If x and y are nonnegative real numbers, then

$$\frac{x+y}{2} \geq \sqrt{xy}.$$

Proof: We assume that x and y are positive real numbers. We want to show it follows that $\frac{x+y}{2} \geq \sqrt{xy}$. Observe that since x and y are real numbers, and $\mathbb{R}$ is closed under subtraction, $(x-y)$ is also a real number. Further, since the square of any real number is greater than or equal to zero, we know that $(x-y)^2 \geq 0$. Expanding the left side of this inequality gives us

$$x^2 - 2xy + y^2 \geq 0.$$

We now add $4x$ to both sides of this inequality. This is done so that the left side will become the square of $(x+y)$. We see that

$$x^2 - 2xy + y^2 + 4xy \geq 4xy,$$

and therefore

$$x^2 + 2xy + y^2 \geq 4xy.$$

Factoring the lefthand side and dividing both sides by 4, we have

$$\frac{(x+y)^2}{4} \geq xy. \tag{1}$$

Since the function $g(x) = \sqrt{x}$ is an increasing function, if $0 \leq a \leq b$, it follows $g(a) \leq g(b)$. Hence, taking the square root of both sides of Inequality (1), the inequality will be preserved. Moreover, note that for any real number, $\sqrt{a^2} = |a|$. Since x and y are both nonnegative, it follows here that $\sqrt{(x+y)^2} = |x+y| = x+y$. Thus, taking square roots on both sides of Inequality (1) yields

$$\frac{x+y}{2} \geq \sqrt{xy},$$

and the theorem has been proved. In particular, we have shown that if x and y are positive real numbers, then

$$\frac{x+y}{2} \geq \sqrt{xy}.$$

□

[1] If the proposition is true, your job is to write a complete proof for the proposition. If it is false, you should provide a counterexample *plus* make reasonable modifications to the stated conjecture so that a new proposition is true. Then, write a complete proof of this new proposition.